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## Notations

We follow ATLAS [23] conventions for naming groups, conjugacy classes, characters and representations.

$H \leq G$	H is a subgroup of G
$H \trianglelefteq G$	H is a normal subgroup of G
$G / H$	Factor group
$\cong$	Isomorphism
$CL(x)$	Conjugacy class of an element x
$C_G(a)$	Centralizer of an element a in a group G
$C_p$	Cyclic group of order p
$D_{2n}$	Dihedral group of order 2n
$S_n$	Symmetric group of degree n
$A_n$	Alternating group of degree n
$H \rtimes K$	The semi-direct product of H and K
$H : K$	The split extension of H and K
$\chi$	A character
$1_S \uparrow^G$	The induced Character from a subgroup S to a group G
$p : q$	The split extension of the cyclic group of order p and the cyclic group of order q which is isomorphic to $C_p \rtimes C_q$ .
$L_n(p)=PSL_n(p)$	The projective special linear group of dimension n over GF(p)
$U_n(p)=PSU_n(p)$	The projective special unitary group
$S_n(p)=PSP_n(p)$	The projective symplectic group
$Sz(8)$	The Suzuki classical group

$M_{11}, M_{12}, M_{22}, M_{23}, M_{24}$	The Mathieu's sporadic finite non-abelian simple groups
$J_1, J_2, J_3, J_4$	The Janko's sporadic finite non-abelian simple groups
$Co_1, Co_2, Co_3$	The Conway's sporadic finite non-abelian simple groups
$Fi_{22}, Fi_{23}, Fi_{24}$	The Fischer's sporadic finite non-abelian simple groups
HS	The Higman-Sims's sporadic finite non-abelian simple groups
McL	The McLaughlin's sporadic finite non-abelian simple groups
Ru	The Rudvalis sporadic finite non-abelian simple group
O'N	The O'nan's sporadic finite non-abelian simple group
Suz	The Suzuki's sporadic finite non-abelian simple group
He	The Held's sporadic finite non-abelian simple group
HN	The Harada-Norton's sporadic finite non-abelian simple group
Ly	The Lyon's sporadic finite non-abelian simple group
Th	The Thompson's sporadic finite non-abelian simple group
B	The Baby Monster sporadic finite non-abelian simple group
M	The Monster sporadic finite non-abelian simple group
$p^{1+2n}$	The extra special group
$p^n$	The elementary abelian group
$M(S)$	The maximal subgroup which contains S
$Q_8$	The quaternion group ( a non-abelian group of order eight)
$(p,q,r)$ -group	That group G generated by two elements , the first one is of order p and the second is of order q and their product is of order r. these two elements can be taken as representatives of conjugacy classes from the character table of G