Introduction

Let p be a prime number. A p-modular system is a triple $(K, \mathcal{O}, F)$ where K is a field of characteristic zero, $\mathcal{O}$ is a complete discrete valuation ring and $F = \mathcal{O}/J(\mathcal{O})$ is an algebraically field of characteristic p, where $J(\mathcal{O})$ means a jacobson radical of $\mathcal{O}$. In this dissertation we will depend on the field F.

In Chapter 1, the first section we present the concept of group action. First we shall define a group action G on a set X. We shall define the following notions: stabilizer, orbit and fixed point of a set. We show that the stabilizer is a subgroup of G. We then recast the proof of Orbit-Stabilizer Theorem. We shall recast the proof of Burnside's Counting Theorem. In the second section, we present the concept of algebra A over the field F, which is very important. We will mention some important examples of algebra. Then we shall define G-algebra over the field F. In the third section, after we have presented G-algebra. Assume that G is a finite group and H is a subgroup of G. We will define G-fixed points A^G of algebra A. We will define the restriction map Res_H^G and the relative trace map t_H^G. We will explain the relative trace map is linear and independent of co-set representative. We will prove the image of the relative trace map is an ideal in the subalgebra of G-fixed points of A. After that we will define the Brauer quotient and Brauer homomorphism. We will give some examples.

In mathematics, especially order theory, there is a very useful concept is partially ordered set. It is defined as a set which follows partial order relation. Let P be a set. The relation $\leq$ defined on a set P is known as partially ordered relation and the set P is called a partially ordered set under certain conditions. This is what we study in Chapter 2, Section 2.1. Also in this section we will study an incidence algebra $I(P, F)$ of the locally finite partially ordered set P over the field F. The idea of the incidence algebra of a locally finite partially ordered set was proposed by G.-C.Rota [11] as the basis for a unified study of combinatorial theory. The study of incidence algebras was continued by Smith. We mean by the concept of the incidence algebra $I(P, F)$ as the set of functions mapping intervals of P to F. We will define the addition, the scalar multiplication and another operation which is called convolution product on $I(P, F)$. We will define incidence functions of the incidence algebra $I(P, F)$ and of which the kronecker delta function, the zeta function and the möbius function. Then we explain properties this functions in terms of their reversibility. The most important question, do all functions in the incidence algebra $I(P, F)$ have an inverse?. Are there conditions for this property?. After that we will discuss when $I(P, F)$ and $I(Q, F)$ are isomorphic, does P and Q isomorphic?. This question has been answered by Stanley [4, 13]. We then recast the proof of the Möbius Inversion Formula which can be seen in [11] or [12]. In Section 2.2, defining the field in this section will be more important to us as the type of representation will be determined by the characteristic of the field and its relationship to the order of G. We consider G as a finite group and P is a locally finite poset of subgroups of G ordered by inclusion. We have two cases for the characteristic p, the first if the characteristic p divides the order $|G|$ and in this case the representation is called a modular representation. In the other case if the characteristic p does not divide the order $|G|$ then the modular representation is completely reducible, as with ordinary representations. We study in this section a new concept which is a modular incidence algebra by present some examples. In Section 2.3, we elucidate that a finite group G acts on a poset P. Then we generalize some results to the notion modular incidence G-algebra. Of these results, a finite group G action on a modular incidence algebra. We explain that the modular incidence G-algebra form an interior G-algebra.

In Chapter 3, the concept of tensor products are presented in this chapter. In Section 3.1, if V and U are vector spaces over the same field F, we can define the product of V and U and denoted by $V \times U$. The product $V \times U$ as a vector space over F. We then mention a theorem to depend a tensor product on it and how to construct a tensor product. In Section 3.2, if A and B are algebras over F, we define a tensor product of A and B and denoted by $A \otimes_F B$ which we will explain it is an algebra over F. If we have two finite groups G_1 and G_2. Consider A and B as G_1-algebra and G_2-algebra respectively we conclude that the tensor product $A \otimes_F B$ is a $G_1 \times G_2$-algebra and we prove that the tensor product $A \otimes_F B$ considered an interior $G_1 \times G_2$-algebra. In Section 3.3, if $(P_1, \leq_1)$ and $(P_2, \leq_2)$ are two posets. We present notion of cartesian product of $(P_1, \leq_1)$ and $(P_2, \leq_2)$ and denoted by $(P_1 \times P_2, \leq)$ which is considered a poset. A poset P may be countable or uncountable. We will deal with an uncountable poset. We will mention that if the posets $(P_1, \leq_1)$ and $(P_2, \leq_2)$ are uncountable then also the cartesian product is uncountable. We can define an incidence algebra of $(P_1 \times P_2, \leq)$ over F and we will learn how to form its elements. If we have two incidence algebras $I(P_1, F)$ and $I(P_2, F)$ then the tensor product $I(P_1, F) \otimes_F I(P_2, F)$ is an incidence algebra. We then prove that the incidence algebra $I(P_1 \times P_2, F)$ is isomorphic to the tensor product of incidence algebra $I(P_1, F) \otimes_F I(P_2, F)$.

In Chapter 4, Section 4.1, since the modular incidence algebra $I(P, F)$ is finite dimensional then we can decomposed it into $e_i I(P, F)$, where e_i is a central primitive idempotent of $I(P, F)$. An algebra $e_i I(P, F)$ is called a block incidence algebra which is an ideal and we prove that. We can decomposed a block incidence algebra into indecomposable $I(P, F)$-module. We define a defect group of a block incidence algebra. In Section 4.2 we will define a concept of a pointed group of the modular incidence algebra and a subgroup of a pointed group. We study some concepts such as a projective relative, a local pointed group, a defect pointed group and a nilpotent block algebra of the modular incidence algebra. We give some examples. In section 4.3 we present a notion of a category $\mathcal{C}$ and give some examples. We define a category algebra $F\mathcal{C}$ of $\mathcal{C}$ over F. Then we connect the two concepts of algebra and incidence algebra together.

We have already written a paper which contains this work in the title: on modular incidence G-algebras. We have already submitted that paper for suitable publication.

Historical Survey

- Gian-Carlo Rota was the first mathematician to study the incidence algebra of locally finite partially ordered sets. In the paper which has the title on the foundations of combinatorial theory I. Theory of Möbius functions published in (1964).
- Richard P.Stanley discussed some results in the incidence algebra. In the paper which has the title structure of incidence algebras and their automorphism groups published in (1970).
- Peter Doubilet, Gian-Carlo Rota and Richard Stanley discussed the main facts on the structure of the incidence algebra of a partially ordered set. In the paper which has the title on the foundations of combinatorial theory (VI): The idea of generating function published in (1972).
- Eugene Spiegel and Christopher J. O'Donnell (1997) studied the incidence algebra and the main topics covered were the maximal and prime

ideals of the incidence algebra, its derivations and isomorphisms, its radicals.

- Ancykutty Joseph studied the incidence algebras and directed graphs. In the paper which has the title on incidence algebras and directed graphs published in (2002).
- Ahmad M. Alghamdi studied the tensor product of incidence algebras. In the paper which has the title tensor product of incidence algebras published in (2014).