Introduction

Claude Shannon [23] initiated the theory of Error-Correcting codes in connection with problems in information theory and coding theory regarding the search of a reliable and efficient transfer of digital information. Linear codes gained more attention from the work of W. Hamming in 1950 [8]. An (n, k, d) code over a field $\mathbb{F}$ with q elements should have a (reasonably) large size in order to encode a large number of source messages and on the other hand should have a relatively large weight (minimum distance) d for detecting and correcting large number of errors that may occur while transmission [21]. There are several types of known codes such as HAMING CODES, HADAMARD CODES, REED-MULLER CODES, REED SOLOMON CODES, BCH CODES and THE GOLAY CODES ,... etc. [19]. It turns out that, for error-correcting properties, the most important types of codes are cyclic codes.

The first connection between codes and group rings of finite groups appeared in the work of F. G. MacWilliams (1969) [17] in which cyclic codes were identified with ideals in the group algebras of cyclic groups, consequently; two sided ideals in $\mathbb{F}G$ are named codes. Since then the algebraic structure of the group ring has been deeply involved in the study and constructions of codes. In particular properties of (central) primitive idempotents in the group algebra of finite groups over finite fields are heavily used in codes construction [2] , [5].

In (2006) T. Hurley [14] (starting with a coding matrix of the finite group G based on an appropriate listing of its elements) proved that the group ring RG of a finite group of order n over a ring R is isomorphic to certain well-defined ring of matrices and hence gave a construction of codes from certain elements of the group ring such as units and zero divisors [11] (his construction was applied to obtain binary codes from the group algebra $\mathbb{F}_2 D_{2k}$). This allows matrix algebras to be used to produce codes by providing generating and check matrices for codes. The coding matrices are known for several classes of groups such as cyclic, elementary-abelian and dihedral groups D_{2k}. In 2018, M. Hamed determined the coding matrices for the general linear group $GL(2, q)$ using its BN-pair structure [6] , [7]. It turns out also that the coding matrix of the direct product group is the tensor (Kronecker) product of the coding matrices of the individual groups in the product deducing the corresponding known result for finite abelian groups.

The aim of this dissertation is to determine the coding matrix of the semi direct product group $G = C_n \times_\phi C_2$; $\phi : C_2 \to Aut(C_n)$ of two cyclic groups in order to generalize the known result for the dihedral group D_{2n} [14], which is known to be a semi direct product of the two cyclic groups C_n, C_2.