

INTRODUCTION

The pseudoblock is the branch of block theory that it partitions the set of classes of indecomposable modules in a very useful way. The origins of pseudoblock were in the paper of Ahmed A. Khammash “The Pseudoblocks of Endomorphism Algebras”, written in 2009, [16]. In this work, he introduced the notion of pseudoblock of the endomorphism algebra $E(Y) = \text{End}_A(Y)$ and showed the compatibility of the pseudoblock distribution of the indecomposable A -summands of Y with the block distribution of the simple $E_A(Y)$ -modules. In 2014, Ahmed A. Khammash [17] studied compatibility between the pseudoblock of endomorphism algebras and the tensor product, and he related the Brauer-Fitting correspondence as well as the notion of pseudoblocks of endomorphism algebras to the tensor product of modules and algebras.

In this dissertation, we borrow the notion “pseudoblock” from [16] to introduce it to finite dimensional (not necessary endomorphism) algebras. We shall investigate the pseudoblock structure of several finite dimensional algebras. The dissertation is organized as follows:

Chapter 0 is a background chapter collects all basic notions and results which are needed for this dissertation.

Chapter 1, we introduce the concept of pseudoblocks of finite dimensional algebras A , and we explain the concept of the Brauer linkage principle of finite dimensional algebras, then we borrow the notion of pseudoblocks of an endomorphism algebra of a module and introduce it for any finite dimensional algebras in the light of the Brauer-Fitting correspondence, where we find that the pseudoblocks for the indecomposable A -summand of Y is compatible with the block for the simple $\text{End}_A(Y)$ -module. Also, we introduce criteria simplifies the determination of the pseudoblock linkage principle.

Chapter 2, we discuss the connection between the Brauer linkage principle \approx_A and the pseudoblock linkage principle \approx_{PSA} , where we find that the pseudoblock linkage principle is stronger than the Brauer linkage principle.

Chapter 3, we revise the concept of tensor product of algebras and modules. Then we prove that the notion of pseudoblocks is compatible with the tensor product. We also study compatibility between the tensor product and the indecomposable A -module, and Brauer linkage principle.

Chapter 4, we discuss the pseudo-block distribution of the indecomposable modules for some various finite dimensional algebras such as semisimple algebras, the triangular algebra A , the symmetric group algebra FS_3 in all characteristics, cyclic group algebra over a field of characteristic prime number p , and p -group algebra.

Chapter 5, we determine the pseudoblock structure of the group algebra of special linear group $\Lambda = FG; G = SL(2, p)$ in characteristic prime number p . We have chosen the group algebra $\Lambda = FSL(2, p)$, because this is the only finite group of Lie type, which is of finite representation type; i.e. Λ has finite indecomposable modules. Furthermore, we introduce the complete set of projective indecomposable Λ -module as stated in [2], and then we describe the complete set of indecomposable Λ -module as stated in the paper of D. Craven [4], where we use the Green Correspondence theory, which relates the isomorphism classes of indecomposable FG -modules with isomorphism classes of indecomposable $FN_G(U)$ -modules; So that we can use theorem (0.3.11). Also, we find the block theory of Λ -modules as stated in [9].

Finally, we determine the pseudoblocks of the group algebra $\Lambda = FSL(2, p)$ in characteristics p , also we study the pseudoblocks of group algebra $FSL(2, p)$ in characteristics $p = 2, 3$ and 5 , and then compare the block and pseudoblock theory of the group algebra Λ in characteristics p .